GRAPHO-ANALYTICAL CALCULATION OF PARTICLE SIZE DISTRIBUTION CHARACTERISTICS OF CONCRETE AGGREGATES

Ph.D. Tibor KAUSAY – Ph.D. Tamás SIMON

The grading of concrete aggregates (sands, sandy gravels, crushed stones, stone dusts etc.) numerically the particle size distribution is characterised namely by the mean value, the standard deviation, the variation coefficient, the average particle size, the fineness modulus and the specific surface area (by volume). The article introduces a united system for the determination of particle size distribution characteristics. The developed grapho-analitic calculation method unites the visualization of graphic methods with the consistence of the analytic methods. The application of the method means the determination of the particle size distribution characteristics by solving simple equations which represent the areas above or under the grading curves which are presented in the appropriate coordinate system. The usability of the equations is greatly helped by computerisation. The figures offer better understanding of the formulas.

Keywords: aggregate, sandy gravel, crushed stone, grading, fineness modulus, specific surface

1. INTRODUCTION

During research, development and high standard design of the aggregates or fillers of concretes, mortars and asphalts, the grading of the sands, sandy gravels, crushed stones and stone dusts can be numerically specified by the mean value, the standard deviation, the variation coefficient, the average particle size, the logarithmic fineness modulus and the specific surface by volume (Kausay, 1975). The determination of them is always based on the result of a sieve or a sedimentation test. The calculation however can be done in two ways that is graphically or analytically, depending on that the area under or above the curve in the coordinate system - proportional to the grading properties - is determined or we describe the values giving the grading properties. The area determination is done basically by measurement while the values are evaluated solely by calculations.

Both methods have differential and also differentia variety.

The developed method carries the signs of grapho-analytical differentia calculus. The method is grapho-analytical, because by area calculation we carry out moment determination,

and differentia calculus, and because for the determination of the grading properties we use directly the results of the sieve or the sedimentation test.

2. THE PRINCIPLE OF THE CALCULATIONS

As a principle of the calculations we use the common property of the grading, that the value of each can be expressed - depending on their characteristics - using a coordinate system which has a suitably transformed x axis by the area above or under the grading curve. For this the scale on the x axis must be chosen in such a manner that in the coordinate system the areas will be proportional to the grading properties, what can be achieved by transforming the originally linear x axis suitably. The linearly scaled x axis is the carrier of the particle size, while the transformed axis carries the descendant of the particle sizes.

At first let us solve the calculation of the area in general because the determination of the grading properties can be derived from the solution of the general form of the area under the grading curve.

In the present paper under the p_δ density function and its derivatives must always be understood a probability density function which is expressing mass proportion, while under the corresponding P_δ distribution function and its derivatives always the relative mass distribution function should be understood even if the scaling of the y axis is in mass percentages.

According to fig. 1. and 2. let us indicate the value of the particle size distribution by P, the boundaries of the subsets by δ, which in case of linearly scaled x axis means d, in case of quadratic scaling d^2, in case of logarithmically scaled x axis lg$d$, while in case of reciprocal x axis scaling would mean d^{-1}. The unit of d is mm. Let the indexes refer to the subset boundaries, to the first the no. *1* and *n* to the last which are the particle bulk's smallest and biggest particle size.

To derive the *general form of the relation* we should take the grading curve as consisting of straight lines and calculate the T_δ area under the P_δ grading curve:

$$T_\delta = \int_{\delta_1}^{\delta_n} P \, \mathrm{d}\delta = \sum_{i=1}^{n\text{-}1} T_i = \frac{1}{100} \cdot \sum_{i=1}^{n-1} \frac{P_i^{\%} + P_{i+1}^{\%}}{2} \cdot (\delta_{i+1} - \delta_i) =$$

$$=\frac{1}{100}\cdot\left[\frac{P_1^{\%}+P_2^{\%}}{2}\cdot(\delta_2-\delta_1)+\frac{P_2^{\%}+P_3^{\%}}{2}\cdot(\delta_3-\delta_2)+\ldots+\frac{P_{n-1}^{\%}+P_n^{\%}}{2}\cdot(\delta_n-\delta_{n-1})\right]=$$

$$=\frac{1}{200}\cdot\left[100\cdot(\delta_n-\delta_{n-1})+\sum_{i=2}^{n-1}P_i^{\%}\cdot(\delta_{i+1}-\delta_{i-1})\right] \quad (1)$$

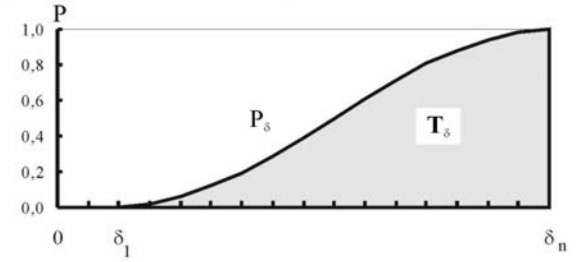


Fig. 1 The T_δ area under the P_δ grading curve

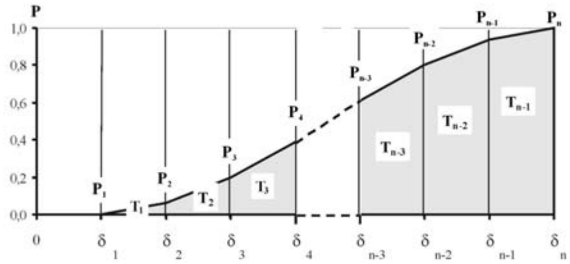


Fig. 2 Division of T_δ area to sub areas

Following we look for the connection between the corresponding areas under the curve and the different grading characteristics.

3. EXPECTED VALUE OR LINEAR FINENESS MODULUS

The expected value or in other words the linear modulus of fineness of the d particle size is equivalent to the area above the grading curve $m_{lin,}$ which is plotted in a coordinate system having a linearly scaled x axis in the interval of $d=0$ and d_n (formula (2) and Fig. 3 and 5.).

$$m_{lin}=\nu_1=\Psi_0=\int_{d_1}^{d_n} d\cdot p\,\mathrm{d}d=\int_{0}^{d_n}\mathrm{d}d-\int_{d_1}^{d_n}P\,\mathrm{d}d=d_n-T_\delta=$$

$$=\frac{1}{200}\cdot\left[100\cdot(d_{n-1}+d_n)+\sum_{i=2}^{n-1}P_i^{\%}\cdot(d_{i-1}-d_{i+1})\right] \qquad [\mathrm{mm}] \quad (2)$$

The T_d expression necessary to express equation (2) can be obtained from equation (1) by inserting $\delta=d$.

The meaning of equation (2) using the signed areas on Fig. 3., 4. and 5 is:

m_{lin} (=) $T_{4.\ Figure}-T_{5.\ Figure}$

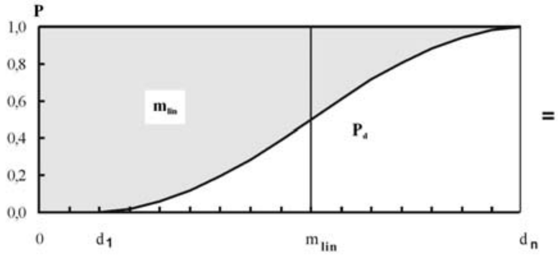
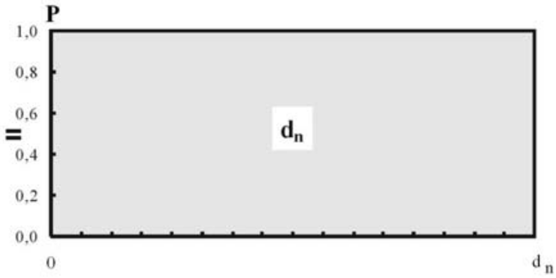


Fig. 3 The area m_{lin} above P_d grading curve



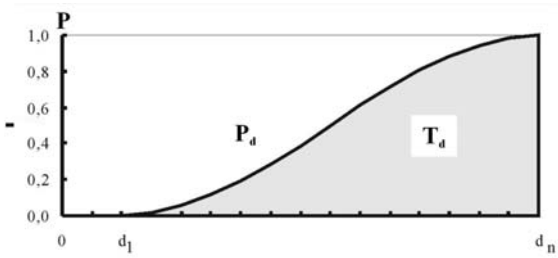


Fig. 4 The area d_n under the line P =1

Fig. 5 The area T_d under P_d grading curve

As an explanation of equation (2) we must indicate that the k^{th} moment ν_k of the area under the p_d density function to the x axis is:

$$\nu_k = \int_{d_1}^{d_n} d^k \cdot p \,\mathrm{d}d \qquad [\mathrm{mm}^k]$$

and the $(k\text{-}1)^{th}$ moment Ψ_{k-1} of the area above P_d distribution function to the x axis (if k is not a negative number):

$$\Psi_{k-1} = \int_0^{d_n} d^{k-1} \cdot (1-P) \,\mathrm{d}d \qquad [\mathrm{mm}^k]$$

From the definition it is derived that ν_0 is nothing else then the area under the density function p_d and its value is $\nu_0=1$ (Fig. 6).

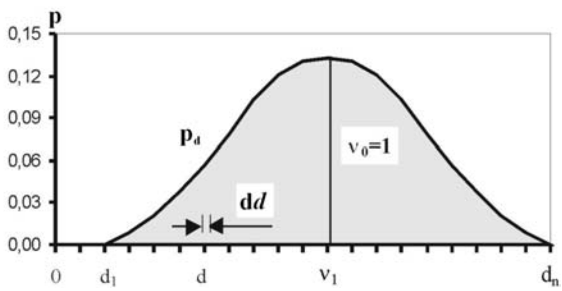


Fig. 6 The ν_0=1 area under the density function p_d

Further, if $k \neq 0$, and if $k=1$ then:

$$\nu_{k\neq 0} = k \cdot \Psi_{k-1} \quad [\mathrm{mm^k}] \qquad \text{és} \qquad \nu_1 = \Psi_0 = m_{lin} \quad [\mathrm{mm}]$$

that is the latter is also the definition of the linear modulus of fineness.

4. THE SQURE OF THE STANDARD DEVIATION

Simplifying we can say that the p_d density function and the corresponding P_d distribution function can be characterized by their expected value and square of the standard deviation. We have seen that the expected value is equal to the moment ν_1 of the area under the p_d density function to the x axis.

The square of the standard deviation is the secondary central moment μ_2 of the area under the density function p_d to the vertical line of the expected value. It can be proved that the square of the standard deviation i.e. the μ_2 central moment is equal to the difference of the squares of the ν_2 second order moment and the ν_1 primary moment. This is nothing else, but the difference of the squares of the double of Ψ_1 primary moment (to the x axis) and of the expected value of the area above the grading curve expressed in a coordinate system having a linear x axis:

$$\sigma^2 = \mu_2 = \nu_2 - \nu_1^2 = 2 \cdot \Psi_1 - m_{lin}^2 \qquad [\mathrm{mm^2}]$$

where in our case using k=2, μ_k in general is the kth order central moment of the area unther the p_d density function to the vertical line of the expected value:

$$\mu_k = \int_{d_1}^{d_n} (d - \nu_1)^k \cdot p \,\mathrm{d}d \qquad [\mathrm{mm^k}]$$

If the P distribution function is presented in a coordinate system which has not a linearly but a quadratically scaled x axis then into the place of the double Ψ_1 primary moment a zero order moment will come into force. That is the square of the standard deviation is equal to the difference of the area (d_n^2-T_{d2}) above the grading curve expressed in a coordinate system having a quadratically scaled x axis in the interval of $d=0$ and d_n^2 and the square of the expected value:

$$\begin{aligned} \sigma^2 &= d_n^2 - T_{d^2} - m_{lin}^2 = \\ &= \frac{1}{200} \cdot \left[100 \cdot (d_{n-1}^2 + d_n^2) + \sum_{i=2}^{n-1} P_i^{\%} \cdot (d_{i-1}^2 - d_{i+1}^2)) \right] - m_{lin}^2 \qquad [\mathrm{mm^2}] \end{aligned} \tag{3}$$

The content of equation (3) expressed by areas can be seen on Fig. 7., 8. and 9.

σ^2 (=) $T_{7.\,Fig} - m_{lin}^2 = T_{8.\,Fig} - T_{9.\,Fig} - m_{lin}^2$

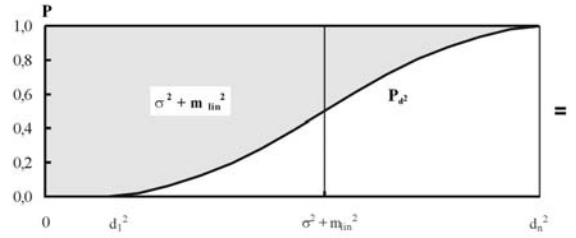


Fig. 7 The area $\sigma^2 + m_{lin}^2$ above the P_d^2 grading curve

Fig. 8 The area d_n^2 under the line $P = 1$

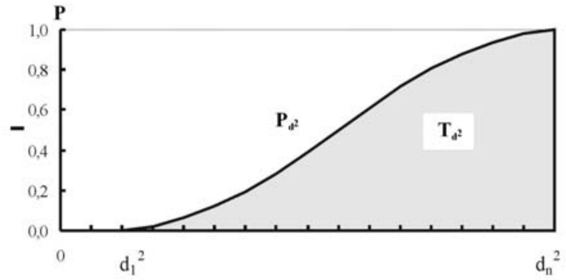


Fig. 9 The area T_d^2 under the P_d^2 grading curve

Using the square of the standard deviation σ^2 can be calculated the standard deviation σ, the relative square of the standard deviation σ^2/m_{lin}^2 and the relative standard deviation σ/m_{lin} that is the coefficient of variation.

5. LOGARITHMIC EXPECTED VALUE AND AVERAGE PARTICLE SIZE

The expected value or in other words the linear fineness modulus of the grading curve is interpreted in a coordinate system which has a linearly scaled x axis. If the grading curve is interpreted in a coordinate system which has logarithmically scaled x axis then instead of the previously discussed expected value we obtain the logarithmic expected value. Since the logarithmic expected value is nothing else then the logarithm to the base ten of the average particle size (lg$d_{average}$), the average size of the particles can be calculated. So the average of the particle size is interpreted in a coordinate system having a logarithmically scaled x axis and its value differs from the value of the linear fineness modulus.

The expected value is expressed by the area above the distribution function by definition. The boundaries of the area above the x axis having a starting point of zero is $d=0$ and d_n which are the y axis and the maximum size of the aggregate.

In case of logarithmically scaled x axis the maximum size of the particles can be found at the lgd_n position, but the interpretation of the x axis is more complicated because the starting point – since lg0=-∞ – must be different from zero. Due to this the role of the y axis is taken by a vertical at lg1=0 value on the x axis, which divides the coordinate system into two parts. Since the logarithm of the numbers smaller than one are less than zero that is negative, the sign of the area left to the lg1=0 point will be negative.

The grading curve of d_1<1 mm smallest particle size depending on the value of the maximum size is located in this negative area (if d_n<1 mm), or extends there (if d_n>1 mm). This later possibility is taken the most general situation in practice.

So in connection of the logarithmic expected value we may not talk about the above the curve area, but about the area boundered by the curve. Accordingly the logarithmic expected value is equal to the area boundered by the distribution function expressed in a coordinate system which has a logarithmically scaled x axis. This area is out of two parts, one is to the left from the x = lg1 = 0 vertical, where the area under the curve is considered, while to the right of this line the area above the curve must be considered (Fig. 10) which can be expressed as follows:

$$\lg d_{average} = \lg d_n - T_{\lg d} =$$
$$= \frac{1}{200} \cdot \left[100 \cdot (\lg d_{n-1} + \lg d_n) + \sum_{i=2}^{n-1} P_i^{\%} \cdot (\lg d_{i-1} - \lg d_{i+1}) \right] \tag{4}$$

In equation (4) the unit of d is mm and the value of lg$d_{average}$ is to be considered as unit less.

Expressing the content of equation (4) by areas on Fig. 10, 11 and 12:

Lg$d_{average}$ (=) $T_{Fig.10.} = T_{Fig.11.} - T_{Fig.12..}$

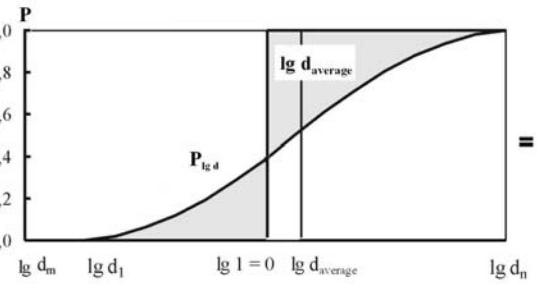


Fig. 10 The area lg$d_{average}$ boundered by the grading curve $P_{\lg d}$

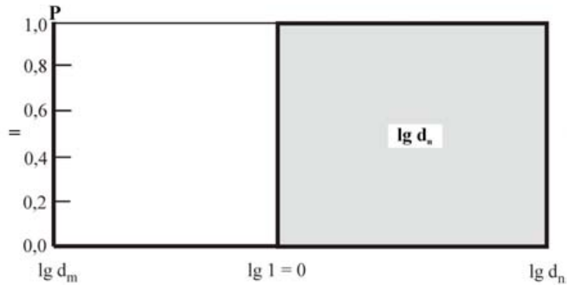


Fig. 11 The area lgd_n under line P =1

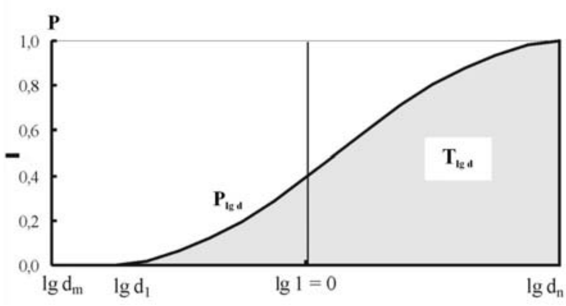


Fig. 12 The area $T_{\lg d}$ under the grading curve $P_{\lg d}$

The average particle size ($d_{average}$) has the value of the logarithmic expected value (lg$d_{average}$) which can be calculated after determining the value of equation (4):

$$d_{average} = 10^{\lg d_{average}} \quad \text{[mm]} \tag{5}$$

Looking at the emphasized area on Fig. 10 it can be seen that lg$d_{average}$ logarithmic expected value and the logarithmic average particle size $d_{average}$ are independent from the starting point lgd_m of the x axis.

6. THE *HUMMEL* AREA AND LOGARITHMIC FINENESS MODULUS

The logarithmic fineness modulus can be calculated from the logarithmic expected value. To derive it, it is necessary to calculate the Hummel area F_{lgd}, which is — according to the classical interpretation — the same as the area above the grading curve which is expressed in a coordinate system with a base ten logarithmic scaled x axis, extending until the lgd_m starting point of the x axis Hummel (1930; 1959.). The procedure can be carried out by taking the difference between the areas represented by lg$d_{average}$ and lgd_m which due to the negative sign of lgd_m is numerically an addition:

$$F_{\lg d} = \lg d_{average} - \lg d_m = m_{\lg} \cdot \lg 2 \tag{6}$$

The unit of $d_{average}$ and d_m is mm, while the value of $F_{\lg d}$ and the logarithmic fineness modulus $m_{\lg}$ are both unitless numbers.

The content of expression (6) by the areas signed in Fig. 13, 14 and 15 is:

$F_{\lg d}$ (=) $T_{Fig.13} = T_{Fig.14} - T_{Fig.15}$

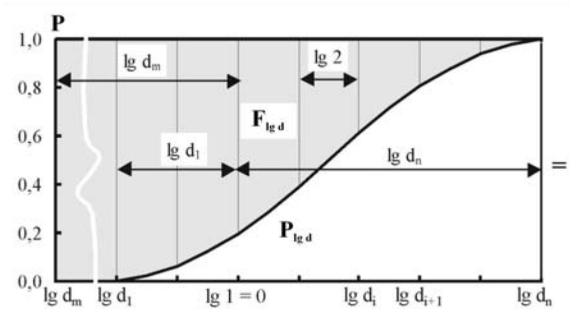


Fig. 13 The Hummel area $F_{\mathrm{lg}d}$ above $P_{\mathrm{lg}d}$ grading curve

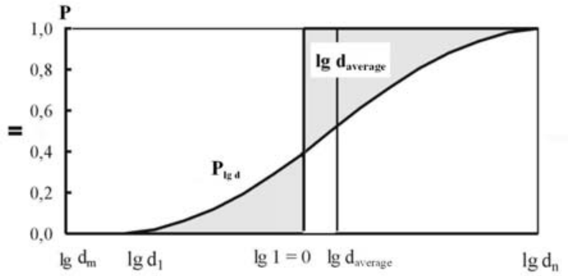


Fig. 14 The area lg$d_{average}$ boundered by the garding curve $P_{\mathrm{lg}d}$

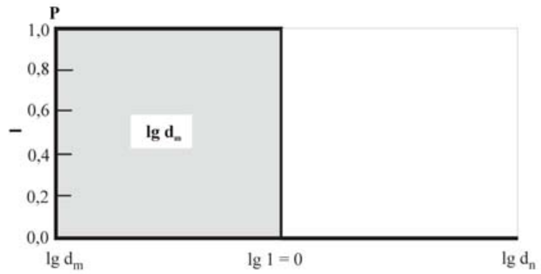


Fig. 15 The area lgd_m under P =1 straight line

On Fig. 13 it can be well seen that the magnitude of the Hummel area and consequently it's derivatives alike *the logarithmic fineness modules* greatly depend on the initial value lgd_m of the x axis, which bounders the Hummel area or the magnitude of the corresponding value of the $d_m \neq 0$ particle size, which is to be agreed upon Popovics (1952; 1953) .

For the examination of the particle size distribution *Abrams* (1918) used the American *Tyler* sieves the characteristics of which is that the finest sieve size — that is the starting point of the x axis — is 0.147 mm and the further sieves have the size always the double of the previous one that is $0.147 \cdot 2^{(i-1)}$, where i = 1, 2, 3,…, n. Taking the logarithms of them the sieve sizes (lg0,147)+(i-1)·(lg2) on a logarithmically scaled x axis would be equally distanced (lg2), a constant value (Fig. 13).

By using this property of the *Tyler* scaled x axis the *Hummel* area (F_{lgd}) can be derived as the sum of partial areas which all have a base length of (lg2) and the height can be calculated from the y values (1-P_i) Fig. 2 and 13. To derive formula (6) we followed this method:

$$F_{\mathrm{lg}d} = \sum_{i=1}^{n-1} \frac{(1-P_i)+(1-P_{i+1})}{2} \cdot \lg 2 = m_{\mathrm{lg}} \cdot \lg 2 \qquad \text{from here:} \qquad m_{\mathrm{lg}} = \frac{F_{\mathrm{lg}d}}{\lg 2} \tag{7.1}$$

which is the logarithmic modulus of fineness, having no unit.

From this the usual practice of characterizing the grading of concrete aggregates differs certainly which is using also the y values (1-P_i) belonging to lg2 base length — that is the

summarized relative retained masses over doubled sieve sizes — taking the height instead of the side of the partial area. Then the qualifying value of the aggregate by the *Hummel* area is:

$$F = \sum_{i=1}^{n}(1-P_i)\cdot \lg 2 = m\cdot \lg 2 \qquad \text{where:} \qquad m = \sum_{i=1}^{n}(1-P_i) \tag{7.2}$$

the fineness modulus as the product qualifying value which generally is simply called the fineness modulus. It's value slightly differs from the theoretical value derived from expression (7.1) but in the practice it is neglected. The fineness modulus has also no unit.

Accordingly the value of the logarithmic fineness modulus and the product qualifying greatly depend on the starting value of the x axis. In Hungary in concrete technology the starting point of the x axis presently in practice and generally if $d_1 \geq d_m$ — due to the lack of other any other condition — is always chosen as d_m=0,063 mm.

7. SPECIFIC SURFACE BY VOLUME AND BY MASS

The specific surface by volume is the total surface area of a unit body volume of the particles, that is the quotient of the sum of the exterior area of the individual particles of the particle bulk and the sum of the volume of the particles, which mostly also contain pores. The analysation of graphic calculation was carried out by *Fáy* and *Kiss* (1961).

The f_V specific surface of a particle set can be expressed in case of spherical or cubic — that is idealised — shaped particles as six times the (-1) order ν_{-1} moment of the area under the p_d distribution function to the y axis:

$$f_V = 6\cdot \nu_{-1} = 6\cdot \int_{d_1}^{d_n} d^{-1}\cdot p\,\mathrm{d}d \qquad [\mathrm{mm}^{-1}]$$

By introducing $u=d^{-1}$ [mm^{-1}] and substituting the corresponding values into the above expression it can be proved that:

$$f_V = 6\cdot \nu_{-1} = 6\cdot \left(\int_{0}^{u_n} \mathrm{d}u + \int_{u_n}^{u_1} P_u\,\mathrm{d}u \right) \qquad \left[\mathrm{mm}^{-1}\right]$$

According to this the specific surface by volume is equal to six times the area under the grading curve which is expressed in a coordinate system having a reciprocally scaled x axis, that is:

$$f_V = 6 \cdot (d_n^{-1} + T_{d^{-1}}) =$$
$$= \frac{3}{100} \cdot \left[100 \cdot (d_{n-1}^{-1} + d_n^{-1}) + \sum_{i=2}^{n-1} P_i^{\%} \cdot (d_{i-1}^{-1} - d_{i+1}^{-1}) \right] \quad \left[\mathrm{mm}^{-1}\right] \tag{8}$$

The content of equation (8) are the marked areas on Fig. 16, 17 and 18:

f_V (=) $6 \cdot \{T_{Fig.\ 16}\} = 6 \cdot \{T_{Fig.\ 17} + T_{Fig.\ 18}\}$

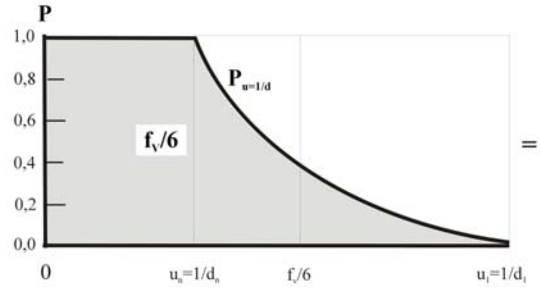


Fig. 16 The area $f_V/6$ under the grading curve $P_{1/d}$

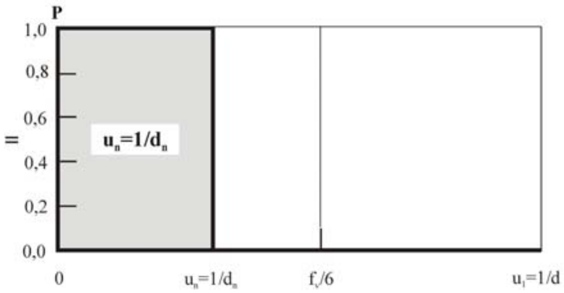


Fig. 17 The area $1/d_n$ under the line P=1

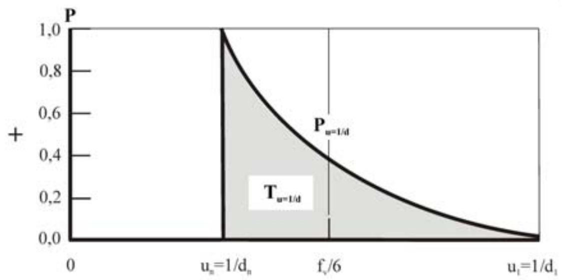


Fig. 18 The area $T_{1/d}$ under the grading curve $P_{1/d}$

The specific surface by volume of the particle bulk is very sensitive to the size and quantity of the fine particles. As an extreme example in case of the bulks having d_1=0 as the smallest particle size the calculation of the specific surface by volume is facing problems, because in this case if d_1=0 then $u_1=\infty$ and $f_V=\infty$, that is the specific surface by volume would be infinite. Due to this reason a condition is usually drawn, which is $d_1 \geq 0{,}001$ mm and $u_1 \leq 1000$ mm^{-1}. Although this condition is arbitrary, the values can be accepted because in the aggregate for concrete the finest particles are the clay particles (which have a particle size less than 0.002 mm), and the quantity of them we limit deliberately.

In construction practice the area of a unit amount of the bulk is more liked to be expressed instead to the volume, by the specific surface by mass and is simply called the specific surface Nischer (1996). The reason of it is simply in the measuring techniques. While the specific surface by volume is a calculated grading property, the specific surface can be measured. The unit of the specific surface is m^2/kg, which is the same as 1000 mm^2/g.

The connection between specific surface f and specific surface by volume is described by the expression (9).

$$f = 10^3 \cdot \frac{f_V}{\rho_T} \qquad [\mathrm{m^2/kg}] \tag{9}$$

where:

f_V = specific surface by volume [1/mm]

ρ_T = the average body density of the particles of the bulk [kg/m^3]

The specific surface also depend on the body density of the material. For this reason — unlikely the specific surface by volume — cannot be considered as a grading property, since the specific surface can only be used directly to compare bulks having the same particle body density.

8. CONCLUSIONS

The advantages of the developed grapho-analytical calculation method of the grading properties are that their principle have a standard structure, makes it possible to use arbitrarily chosen sieve sizes for the sieve tests and to examine arbitrary particle sizes during sedimentation tests. It makes it unnecessary to derive the grading function, eliminates the necessity of area or the proportional distance measurement and actually does not really require the compilation of the grading curve which belongs to the total examination. The application of the calculation method is simple, by solving equations of similar form which can be computerized. The standardisation of the method in Europe is not expected, in Hungary however a standard deals with it which has been accepted and it's sign is MSZ 18288-5:1981. Hopefully this standard will remain in force since no other may substitute it.

9. MOST IMPORTANT NOTATIONS

d	nominal particle size, independent variable
$d_{average}$	average particle size in case of logarithmically calibrated x axis, the lg$d_{average}$ is the number of the logarithmic expected value
d_i	ith particle subset size limit
d_m	value of the starting point of the lgd_m x axis, the corresponding particle size is for the calculation of the logarithmic fineness modulus
f	specific surface (by mass)

f_V specific surface by volume

F practical qualifying value of the *Hummel* area

$F_{\lg d}$ the *Hummel* area

k order of moment of the area under a function

$\lg d_{average}$ the logarithm to the base ten of the average particle size, the logarithmic expected value

$\lg d_m$ value of the starting point of the logarithmically scaled x axis

m the practical product qualifying value of the logarithmic fineness modulus

$m_{\lg}$ the logarithmic fineness modulus

m_{lin} expected value, in other words the linear fineness modulus

n the number of the examined particle sizes

p_d the density function

p_δ the density function transformed according to δ

P_d the distribution function, in other words the grading curve

P_δ the distribution function or grading curve transformed according to δ

P_i distribution function value belonging to the i^{th} particle subset boundary

$T_{Fig.\,z}$ area marked on Fig. z

T_δ area under the transformed distribution function P_δ

δ the value of d particle size transformed proportionally with the grading properties

μ_2 the second order central moment of the area under p_δ density function to the vertical line of the expected value

μ_k the k^{th} order central moment of the area under p_δ density function to the vertical line of the expected value

$v_0 = 1$ area under the density function p_δ

v_{-1} the $(-1)^{th}$ order moment of the area under the density function p_δ to the y axis

$v_1 = \Psi_0$ the first order moment of the area under the density function p_δ to the y axis, which is equal to the area above the distribution function P_δ

v_2 the second order moment of the area under the density function p_δ to the y axis

v_k the k^{th} order moment of the area under the density function p_δ to the y axis

ρ_T average particle body density of dry particle bulk

σ standard deviation

σ^2 square of standard deviation

σ/m_{lin} coefficient of variation

σ^2/m_{lin}^2 the relative standard deviation square

Ψ_1 first order moment of the area above the distribution function P_δ to the y axis

Ψ_{k-1} the $(k-1)^{th}$ order moment of the area above the distribution function P_δ to the y axis (if k is positive)

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